Surface Area:Volume Ratio

Cottontail: by David Iliff CC-BY-SA_3.0 http://www.mammalweb.org/index.php?option=com_content&view=category&id=13&Itemid=124

Snowshoe Hare: By D. Gordon E. Robertson - Own work, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=24071225

Jack Rabbit: By Ryan Hagerty - http://www.public-domain-image.com/public-domain-images-pictures-free-stock-photos/fauna-animals-public-domain-images-pictures/bunny-rabbit-public-domain-images-pictures/side-view-close-up-of-rabbit-sitting-on-gravel-under-brush.jpg, Public Domain, https://commons.wikimedia.org/w/index.php?curid=24837834

Rabbits and hares are in an evolutionary arms race with their predators, such as owls, that is called the **Red Queen hypothesis**. Predators and prey are linked by evolutionary pressures. Predators are constantly evolving to become better hunters while their prey are constantly evolving better defenses against predators. So, for example, owls evolve to become quieter and quiter fliers while rabbits evolve bigger and bigger ears to hear them.

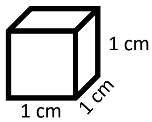
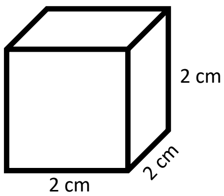
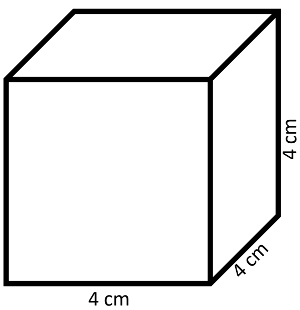
Barred Owl: By L. Green

However, organisms must also evolve to their environment. As a result rabbit ears show differences due to their habitat. The cottontail rabbit lives in a temperate climate that doesn't have severe winters. The snowshoe hare lives in areas with severe winters. Its smaller ears compared to a cottontail reduce heat loss without a substantial decrease in hearing ability. The jack rabbit, on the other hand, ia a desert dweller. It's significantly larger ears act as heat radiators providing a method to cool its body in the hot desert environment.

The relationship between ear size and habitat shows the importance of the ratio between surface area and volume. Snowshoe hares have reduced the surface area of their ears to maintain their heat, while jack rabbits have increased their surface area to dissipate heat. Their body heat is directly related to their volume and in this example remains relatively constant. But what happens when the volume changes, like when organisms grow, or evolve to be bigger? Examining how the surface area:volume ratio changes with size explains some basic biological processes such as binary fission in bacteria, mitosis in eukaryotic cells and why large organism like us have so many organ systems. But first we need to understand the math behind the phenomena.

Biology 11
Evolution 2

Complete the following table (objects may not be to scale). Use the first row as an example:

Cube Dimensions	**Surface Area** SA = 6(l × w)	**Volume** V = l × w × h	**Surface Area:Volume**
	SA = 6(1 × 1) SA = 6 cm^2	V = 1 × 1 × 1 V = 1 cm^3	SA/V = 6/1 SA/V = 6
	SA = 6(2 × 2) SA = 24 cm^2 or SA = 6 × 2^2 = 24	V = 2 × 2 × 2 V = 8 cm^3 or V = 1 × 2^3 = 8	SA/V = 24/8 SA/V = 3 or SA/V = 6 × $(\frac{2^2}{2^3})$ SA/V = 6 × $\frac{1}{2}$ = 3
	SA = 6(4 × 4) SA = 96 cm^2 or SA = 24 × 2^2 = 96	V = 4 × 4 × 4 V = 64 cm^3 or V = 8 × 2^3 = 64	SA/V = 96/64 SA/V = 1.5 or SA/V = 3 × $\frac{1}{2}$ = 1.5
8 cm × 8 cm × 8 cm	96 × 2^2 = 384 cm^2	64 × 2^3 = 512 cm^3	SA/V = 1.5 × $\frac{1}{2}$ = .75

As the cube doubles in size, what is the factor increase in the surface area?

As the cube doubles in size, the surface area doubles to the power of 2 times, so the surface area increases by a factor of 4.

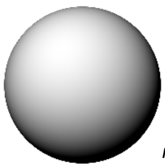
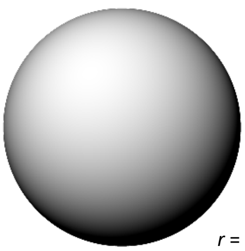
As the cube doubles in size, what is the factor increase in the volume?

As the cube doubles in size, the volume doubles to the power of 3 times, so the surface area increases by a factor of 8.

Why would this be important in biology? (Remember that all organisms receive into their cells, their volume, essential requirements for life, like water, oxygen and nutrients, across their cell membranes, their surface area.)

As a cell gets larger, the volume increases to the power of 3 compared to the surface area increasing to the power of 2. This means that the parts of the cell that require food are getting more and more and more of them while the space on the cell to get food only increases by more and more. Eventually, either all parts of the cell need to get less, or some parts of the cell get a lot less or nothing and die. So, cells can only get to be so big. And that's why cells are so small. The biggest cells are eggs, which contain all the nutrients they need, except for oxygen. Even then, the ostrich which lays the largest eggs, have eggs that are the smallest relative to their body size, because even eggs can only get so big.

Let's try this again with a different shape.

Sphere Dimensions	**Surface Area** SA = $4\pi r^2$	**Volume** $V = \frac{4}{3}\pi r^3$	**Surface Area:Volume**
 r = 1 cm	SA = $4\pi(1^2)$ SA ≈ 12.6 cm^2	V = $\frac{4}{3}\pi \times 1^3$ V ≈ 4.2 cm^3	SA/V = 12.6/4.2 A/V = 3
 r = 2 cm	SA = $4\pi(2^2)$ SA ≈ 50.3 cm^2	V = $\frac{4}{3}\pi \times 2^3$ V ≈ 33.5 cm^3	SA/V = 50.3/33.5 A/V = 1.5
 r = 4 cm	SA = $4\pi(4^2)$ SA ≈ 201.1 cm^2	V = $\frac{4}{3}\pi \times 4^3$ V ≈ 268.1 cm^3	SA/V = 50.3/33.5 A/V = 0.75

Are the results the same? Explain.

Yes. As the radius doubles, the surface area increases by double squared and the volume increases by double cubed. Therefore, the SA/V ratio changes as it did for the cube. However, the first step seems to be missing, but that is because we are using radius rather than diameter. If we started with *d* = 1 cm, which is *r* = 0.5 cm, the SA/V ratios would be identical.

How could this be the driving evolutionary force for binary fission (bacteria dividing into two), mitosis (cell division in eukaryotes) and the evolution of organ systems?

Bacteria divide when they grow to the limit they require to maintain a surface-area-to-volume ratio that is large enough to provide all their cellular functions with the nutrients needed from outside their cell. Eukaryotic cells can grow larger because they have membrane bound organelles like vacuoles and cytosomes that can store nutrients until needed, allowing for a smaller surface-area-to-volume ration than bacteria. However, they eventually grow too big initiating mitosis.

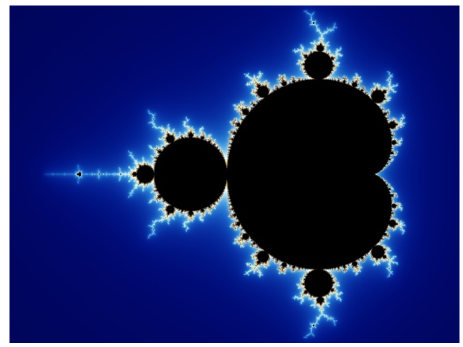
In large organisms, body systems evolved to bring the outside to the inside. Body structures like the microvilli on the cells of our digestive system have also evolved to increase the surface-area-to-volume ratio.

What about Sponges?

Sponges (phylum Porifera) are animals, but are not organized at the tissue level, organ level or the organ system level. Yet, sponges can be large. I have seen orange elephant ear sponges (*Agelas clathrodes*) much larger than me when diving in the Caribbean. So how do sponges overcome the surface-area-to-volume ratio constraints on size (the larger you get the less surface area to supply what you need)? To understand this, we need to look at another branch of mathematics, fractals.

Fractals are geometric objects that appear self-similar at increasing smaller scales. The Ediacaran biota is also described as being fractal-like. Sponges predate the Ediacarans, so it is not surprising that fractals might explain something about their body.

Graph of the Mandelbrot set showing the self-similarity of fractals

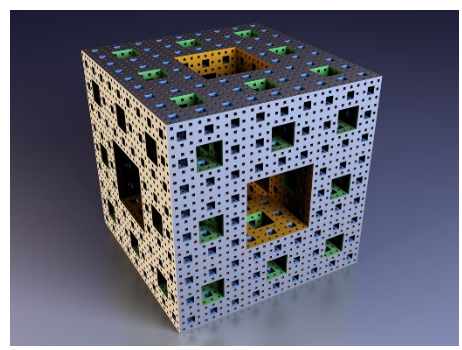


By Created by Wolfgang Beyer with the program Ultra Fractal 3. - Own work, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=321973

Sponges are, like all life on our planet, no matter how flat, 3-dimensional beings. To examine their structure, we will look at a 3-dimensional fractal called a Menger sponge after Karl Menger who first described it in 1926. Each self-similar level in fractals is called an **iteration**. At right bottom is a diagram of a four iteration Menger sponge. The reasons biologists believe early animals had a fractal design is because of the small number of directions, or genes in the case of organisms, that are required to make it. A Menger sponge requires four instructions:

1. Make a cube;
2. Divide each face into nine like a Rubik's cube;
3. Remove the centre cube in each face, and the smaller cube in the middle of the bigger cube;
4. Repeat steps 2 and 3. Each repetition is called an **iteration**.

A four-iteration Menger sponge



By Niabot - Own work, CC BY 3.0, https://commons.wikimedia.org/w/index.php?curid=7818920

Mathematically, if you continue this process for a large number of iterations, you end up with a 3-dimensional shape with almost **infinite surface area** and **zero volume** resulting in a huge SA/V ratio, exactly the opposite of what happens with the SA/V ratio we have seen in non-fractal cubes 2 pages above. Sponges evolved a similar structure. They are an organism full of holes: the ostia, the chambers, the tubes and the spongocoel. This allows the environment to flow through the organism, so the environment touches almost every cell in the sponge's body. Instead of requiring the organism to ingest or inhale parts of the environment into the body, then transferring those particles throughout the body, a sponge draws the environment through its body and to its cells by using the cilia on its choanocytes. It was evolution's first strategy to defeat the size constraints that imposed small size on organisms for over 3 billion years. Although successful, sponges' design restricts it to certain habitats. The eventual evolution of organ systems also defeated the size constraints and enabled diversity allowing organisms that had them to inhabit every habitat on Earth.